MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

ETN3096 – DIGITAL SIGNAL PROCESSING
(CE, EE, LE, OPE, TE)

7 MARCH 2019
2:30 PM – 4:30 PM
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 12 pages (including this cover page) with 4 Questions only and an appendix.
2. Attempt **ALL** questions. The distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.

Question 1

(a) Consider a discrete-time system $y[n] = 5n^3 x[n]$.

- (i) Determine if the system is linear. [5 marks]
- (ii) Determine if the system is time-invariant. [5 marks]

(b) In the system shown in Figure Q1, $h_1[n] = [\ldots\ 0\ \ \underline{1}\ \ 2\ \ 1\ \ 0\ \ldots]$.

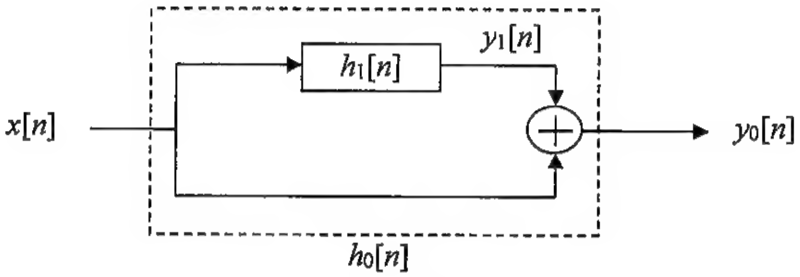


Figure Q1

- (i) For input $x[n] = [\ldots\ 0\ \ \underline{4}\ \ 3\ \ 2\ \ 0\ \ldots]$, determine $y_1[n]$ and $y_0[n]$ for $n = 0$ to 5. [10 marks]
- (ii) Determine the impulse response of the overall system $h_0[n]$. State if it is causal and justify your answer. [5 marks]

Question 2

(a) Given that $H(z) = \dfrac{1}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$, $\frac{1}{3} < |z| < \frac{1}{2}$, do the following:

- (i) Find the inverse z-transform of $H(z)$. [6 marks]
- (ii) If $H(z)$ represents a transfer function, determine the corresponding linear constant coefficient difference equation. [4 marks]

(b) (i) Given a sequence $x[n] = [\underline{1}\ \ 0\ \ 0\ \ 2]$, evaluate the 4-point Discrete Fourier Transform (DFT) of the sequence $x[n]$. Compute all four DFT samples. [6 marks]

(ii) Let $w[n]$ be the circularly shifted signal of $x[n]$ where $w[n] = x[n-2]_4$. Sketch the signal $w[n]$ and compute the 4-point DFT of $w[n]$ by using the **properties of DFT**. All the 4 DFT samples must be computed. [9 marks]

Continued ...

Question 3

(a) Design a 3-tap Finite Impulse Response (FIR) lowpass filter with a cutoff frequency of **1500 Hz** and a sampling rate of **8000 sample/s**.

(i) Determine the filter coefficients of the filter using a *rectangular* window function. [9 marks]

(ii) Write down the transfer function and difference equation of the filter. [3 marks]

(b) A lowpass FIR filter has the following specifications:
Passband: 0 – 800 Hz
Stopband: 1200 – 4000 Hz
Stopband attenuation: 40 dB
Passband ripple: 0.1 dB
Sampling rate: 8000 Hz
Determine the window method, the FIR filter length, and the cutoff frequency to be used in the design. [6 marks]

Continued ...

(c) Figure Q3 shows a simple noise canceller with the use of one-tap adaptive filter. The adaptive filter equation is given in (1) and weight update equation for the least mean square (LMS) algorithm is given in (2).

(i) Describe briefly the algorithm used in adaptive filtering. [7 marks]

(ii) Explain the difference between adaptive FIR filter and non-adaptive FIR filter. [3 marks]

(iii) Complete Table Q3 for $n = 0$ and 1. Calculation steps must be shown. [5 marks]

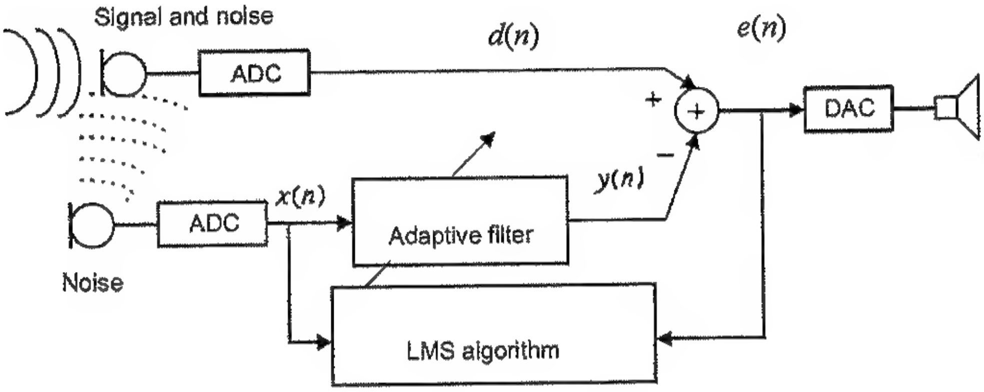


Figure Q3

$$y(n) = w(n)x(n) \quad (1)$$

$$w(n+1) = w(n) + 0.02\, e(n)\, x(n) \quad (2)$$

Table Q3: Adaptive Filter Computation

Iteration n	Signal corrupted with noise $d(n)$	Noise signal $x(n)$	Filter output $y(n)$	Error signal $e(n)$	Filter weight $w(n)$
0	-1.310	-0.150			0.200
1	1.221	0.212			

Continued ...

Question 4

Sketch the structures of an Infinite Impulse Response (IIR) filter with the transfer function

$$H(z) = \frac{0.4\left(1 - z^{-1}\right)}{\left(1 + 0.4z^{-1}\right)\left(1 + 0.8z^{-1}\right)}$$

in the following forms:

(i) Direct Form I [6 marks]

(ii) Direct Form II [5 marks]

(iii) Cascade Form [6 marks]

Continued ...

APPENDIX

Discrete-time Fourier transform

Properties

Property	$x[n], y[n]$	$X(e^{jw}), Y(e^{jw})$
Linearity	$ax[n] + by[n]$	$aX(e^{jw}) + bY(e^{jw})$
Time shifting	$x[n-n_0]$	$e^{-jwn_0}X(e^{jw})$
Frequency shifting	$e^{jw_0 n}x[n]$	$X(e^{j(w-w_0)})$
Time reversal	$x[-n]$	$X(e^{-jw})$
Differentiation	$n^k x[n]$	$(j)^k \frac{d^k}{dw^k}X(e^{jw})$
Convolution	$x[n] * y[n]$	$X(e^{jw})Y(e^{jw})$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}(X(e^{j\theta}) * Y(e^{jw}))$

Common pairs

$x[n]$	$X(e^{jw})$
$\delta[n]$	1
$\delta[n-n_0]$	e^{-jwn_0}
1	$\sum_{k=-\infty}^{\infty} 2\pi\delta(w+2\pi k)$
$e^{jw_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(w-w_0+2\pi k)$
$u[n]$	$\frac{1}{1-e^{-jw}} + \sum_{k=-\infty}^{\infty} \pi\delta(w+2\pi k)$
$a^n u[n],\ \lvert a\rvert < 1$	$\frac{1}{1-ae^{-jw}}$
$(n+1)a^n u[n],\ \lvert a\rvert < 1$	$\frac{1}{(1-ae^{-jw})^2}$
$\frac{\sin w_c n}{\pi n}$	$X(e^{jw}) = \begin{cases} 1, & \lvert w\rvert < w_c \\ 0, & w_c < \lvert w\rvert \leq \pi \end{cases}$
$x[n] = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin w(M+1)/2}{\sin w/2} e^{-jwM/2}$

z-transform

Properties

Properties	Sequence	z-transform	ROC
	$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$	R_x R_{x1} R_{x2}
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z)+bX_2(z)$	Contains $R_{x1} \cap R_{x2}$
Time shifting	$x[n-m]$	$z^{-m} X(z)$	R_x except for the possible addition or deletion of the origin or infinity.
Multiplication by an exponential sequence	$a^n x[n]$	$X(z/a)$	$\|a\|R_x$
Differentiation	$nx[n]$	$-z\frac{dX(z)}{dz}$	R_x except for the possible addition or deletion of the origin or infinity.
Conjugate	$x^*[n]$	$X^*(z^*)$	R_x
Time reversal	$x[-n]$	$X(z^{-1})$	$1/R_x$
Convolution	$x_1[n] * x_2[n]$	$X_1(z)\, X_2(z)$	Contains $R_{x1} \cap R_{x2}$

Common pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\| > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\| < 1$
$\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\| > \|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\| < \|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| > \|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\| < \|a\|$

Finite Impulse Response (FIR) Filters

Ideal impulse responses for standard FIR filters

Filter Type	Ideal Impulse Response $h(n)$
Lowpass:	$h(n)=\begin{cases}\dfrac{\Omega_c}{\pi}, & n=0\\ \dfrac{\sin(\Omega_c n)}{n\pi}, & n\neq 0\end{cases}$
Highpass:	$h(n)=\begin{cases}\dfrac{\pi-\Omega_c}{\pi}, & n=0\\ \dfrac{-\sin(\Omega_c n)}{n\pi}, & n\neq 0\end{cases}$
Bandpass:	$h(n)=\begin{cases}\dfrac{\Omega_H-\Omega_L}{\pi}, & n=0\\ \dfrac{\sin(\Omega_H n)-\sin(\Omega_L n)}{n\pi}, & n\neq 0\end{cases}$
Bandstop:	$h(n)=\begin{cases}\dfrac{\pi-(\Omega_H-\Omega_L)}{\pi}, & n=0\\ \dfrac{-\sin(\Omega_H n)+\sin(\Omega_L n)}{n\pi}, & n\neq 0\end{cases}$

FIR filter length estimation using window functions

Window Type	**Window Function** $w(n), -M \le n \le M$
Rectangular	1
Hanning	$0.5+0.5\cos\left(\dfrac{n\pi}{M}\right)$
Hamming	$0.54+0.46\cos\left(\dfrac{n\pi}{M}\right)$
Blackman	$0.42+0.5\cos\left(\dfrac{n\pi}{M}\right)+0.08\cos\left(\dfrac{2n\pi}{M}\right)$

Normalized Transition Width $\Delta f=\dfrac{\lvert f_{stop}-f_{pass}\rvert}{f_{sampling}}$			
Type of Window	**Window Length** N	**Stopband Attenuation (dB)**	**Passband Ripple (dB)**
Rectangular	$N=0.9/\Delta f$	21	0.7416
Hanning	$N=3.1/\Delta f$	44	0.0546
Hamming	$N=3.3/\Delta f$	53	0.0194
Blackman	$N=5.5/\Delta f$	74	0.0017

Bilinear Transformation (BLT)

1. Frequency prewarping

Let ω_a denote the analog frequency marked on the $j\omega$-axis on the s-plane, and ω_d denote the digital frequency marked on the unit circle in the z-plane.

For the lowpass filter and highpass filter:

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right)$$

For the bandpass filter and bandstop filter:

$$\omega_{al} = \frac{2}{T}\tan\left(\frac{\omega_l T}{2}\right),\ \omega_{ah} = \frac{2}{T}\tan\left(\frac{\omega_h T}{2}\right)$$

and $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}, W = \omega_{ah} - \omega_{al}$.

2. Prototype transformation using the lowpass prototype $H_P(s)$

From lowpass to lowpass: $H(s) = H_P(s)|_{s=\frac{s}{\omega_a}}$

From lowpass to highpass: $H(s) = H_P(s)|_{s=\frac{\omega_a}{s}}$

From lowpass to bandpass: $H(s) = H_P(s)|_{s=\frac{s^2+\omega_0^2}{sW}}$

From lowpass to bandstop: $H(s) = H_P(s)|_{s=\frac{sW}{s^2+\omega_0^2}}$

where ω_a denotes the analog frequency, $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}, W = \omega_{ah} - \omega_{al}$.

3. Substitute the BLT to obtain the digital filter

$$H(z) = H(s)|_{s=\frac{2}{T}\frac{z-1}{z+1}}$$

Conversion from Analog Filter Specifications to Lowpass Prototype Specifications

Analog Filter Specifications	Lowpass Prototype Specifications
Lowpass: ω_{ap}, ω_{as}	$v_s = \dfrac{\omega_{as}}{\omega_{ap}}$
Highpass: ω_{ap}, ω_{as}	$v_s = \dfrac{\omega_{ap}}{\omega_{as}}$
Bandpass: $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$ $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$v_s = \dfrac{\omega_{ash} - \omega_{asl}}{\omega_{aph} - \omega_{apl}}$
Bandstop: $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$ $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$v_s = \dfrac{\omega_{aph} - \omega_{apl}}{\omega_{ash} - \omega_{asl}}$

ω_{ap}, passband frequency edge; ω_{as}, stopband frequency edge

ω_{apl}, lower cutoff frequency in passband; ω_{aph}, upper cutoff frequency in passband

ω_{asl}, lower cutoff frequency in stopband; ω_{ash}, upper cutoff frequency in stopband

ω_0, geometric center frequency

Closed-Form Expression for Some Useful Series

$\sum_{n=0}^{N-1} a^n = \dfrac{1-a^N}{1-a}$	$\sum_{n=0}^{\infty} na^n = \dfrac{a}{(1-a)^2}$
$\sum_{n=0}^{N-1} na^n = \dfrac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$	$\sum_{n=0}^{N-1} n^2 = \dfrac{1}{6}N(N-1)(2N-1)$
$\sum_{n=0}^{N-1} n = \dfrac{1}{2}N(N-1)$	$\sum_{n=N_1}^{N_2} a^n = \dfrac{a^{N_1-1} - a^{N_2}}{1-a}$
$\sum_{n=0}^{\infty} a^n = \dfrac{1}{1-a} \qquad \lvert a\rvert < 1$	

Digital Butterworth and Chebyshev Filter Designs

With the given passband ripple A_p dB at the normalized passband frequency edge $v_p = 1$, and the stopband attenuation A_s dB at the normalized stopband frequency edge v_s, ε is the absolute ripple specification

$$\varepsilon^2 = 10^{0.1A_p} - 1$$

Butterworth lowpass prototype order

$$n \geq \frac{\log_{10}\left(\frac{10^{0.1A_s} - 1}{\varepsilon^2}\right)}{|2\log_{10}(v_s)|}$$

Chebyshev lowpass prototype order

$$n \geq \frac{\cosh^{-1}\left(\sqrt{\frac{10^{0.1A_s} - 1}{\varepsilon^2}}\right)}{\cosh^{-1}(v_s)}$$

where $\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$

3-dB Butterworth Lowpass Prototype Transfer Functions ($\varepsilon = 1$)

n	$H_P(s)$
1	$\frac{1}{s+1}$
2	$\frac{1}{s^2 + 1.4142s + 1}$
3	$\frac{1}{s^3 + 2s^2 + 2s + 1}$

Chebyshev Lowpass Prototype Transfer Functions with 1dB Ripple ($\varepsilon = 0.5088$)

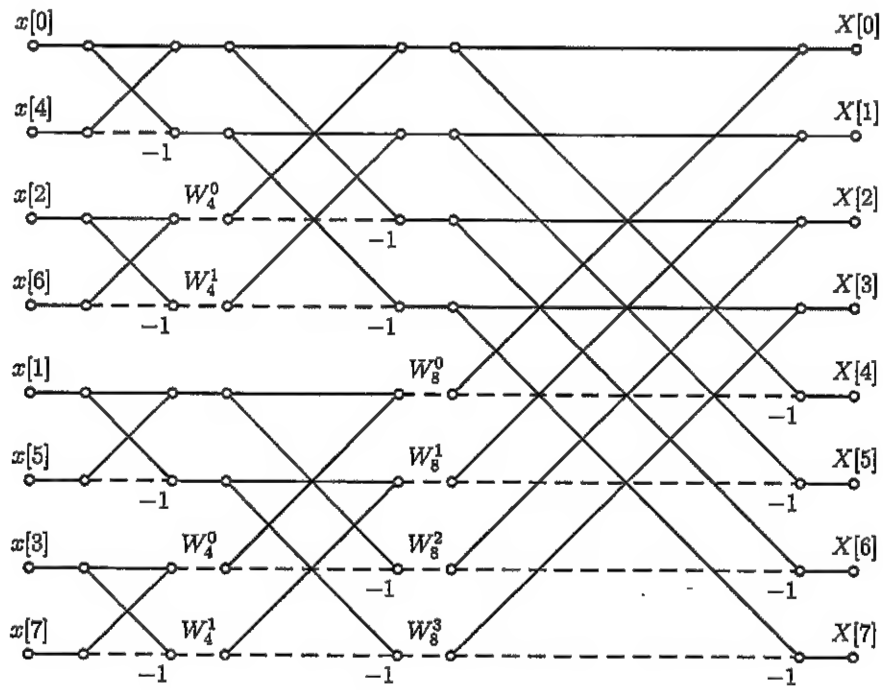
n	$H_P(s)$
1	$\frac{1.9652}{s + 1.9652}$
2	$\frac{0.9826}{s^2 + 1.0977s + 1.1025}$
3	$\frac{0.4913}{s^3 + 0.9883s^2 + 1.2384s + 0.4913}$

Discrete Fourier Transform

Properties

Property	$x[n]$	$X[k]$
Linearity	$A_1x_1[n] + A_2x_2[n]$	$A_1X_1[k] + A_2X_2[k]$
Time shifting	$x[\langle n - n_0\rangle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0n}$	$X[\langle k - k_0\rangle_N]$
Time reversal	$x[\langle -n\rangle_N]$	$X[\langle -k\rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k\rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k]Y[k]$
Modulation	$Nx[n]y[n]$	$X[k] \circledast Y[k]$

The decimation-in-time fast Fourier transform



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